Name - Prabhjot Rani

Class - CSE 3rd Sem. (Lect)

Roll No. - 2105005

Subject - Mathematics III

PART-A

1Q Find the C.F of $y'' - 2y' + y = 0$

Sol The given differential eq is

$$(D^2 - 2D + 1)y = 0$$

A.F is $m^2 - 2m + 1 = 0$

$$m^2 - m - m + 1 = 0$$

$$m(m-1) - 1(m-1) = 0$$

$$(m-1)(m-1) = 0$$

$$m = 1, 1$$

C.S is $y = e^{x}(c_1 + c_2 x)$

Q Define Cauchy's linear equation and also write its symbolic form.

Ans A Linear equation of the form

$$P_0 x^n \frac{d^n y}{dx^n} + P_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots + P_n y = Q(x) \quad \text{--- (1)}$$

where $P_0, P_1 \ldots\ldots P_n$ are real constant and $Q(x)$ is a function of x is Cauchy's linear equation. The equation (1) can be written as in symbolic form $(P_0 x^n D^n + P_1 x^{n-1} D^{n-1} + \ldots\ldots P_n) y = Q(x)$

where $D = \frac{d}{dx}$, $D^n = \frac{d^n}{dx^n}$

$x^2 D^2 + xD + y = 0$

Put $x = e^z$, $z = \log x$

$xD = \theta$, $x^2 D^2 = \theta(\theta - 1)$

$x^3 D^3 = \theta(\theta - 1)(\theta - 2)$

3(Q) Define 'Limit of a function of two variables'

Ans Limit of a function of Two variables

A function $f(x, y)$ is said to tend to a limit l as the point (x, y) tends to point (a, b), corresponding to any pre-assigned positive number $\varepsilon > 0$, however small, we can find a number δ, such that

$|f(x, y) - l| < \varepsilon$ for all point (x, y) other than (a, b) for which

$|x-a| < \delta$ and $|y-b| < \delta$

Note:- It is very important to note that if $\underset{(x,y)\to(a,b)}{Lt} f(x,y)$ exists; then this limit is independent of the path along which we approach the point (a,b)

Note:- $\underset{(x,y)\to(a,b)}{Lt} f(x,y)$, if it exists, is unique.

Q4 Evaluate $\int_1^2 \int_1^3 dx\, dy$

$$= \int_1^2 \left(\int_1^3 dx \right) dy = \int_1^2 [x]_1^3 \, dy$$

$$= \int_1^2 (3-1)\, dy = 2 \int_1^2 dy$$

$$= 2[y]_1^2 = 2(2-1)$$

$$= 2 \times 1 = 2$$

PART-B

Q Find the particular integral (P.I) of

$$y''' - y'' + 4y' - 4y = \sin 3x$$

Sol Given eq is $y''' - y'' + 4y' - 4y = \sin 3x$

The S.F is $(D^3 - D^2 + 4D - 4)\, y = \sin 3x$

$$P.I = \frac{1}{D^3 - D^2 + 4D - 4} \sin 3x = \frac{1}{-9D + 9 + 4D - 4} \sin 3x$$

$$= \frac{1}{-5D + 5} \sin 3x = \frac{1}{5(1-D)} \sin 3x \quad D^2 = -9$$

$$= \frac{1+D}{5(1-D^2)} \sin 3x = \frac{1+D}{5(1+9)} \sin 3x$$

$$= \frac{1}{50}\left[\sin 3x + D(\sin 3x)\right]$$

$$= \frac{1}{50}\left[\sin 3x + 3\cos 3x\right]$$

Q2 if $z = \log(u^2 + v)$, $u = e^{x^2+y^2}$, $v = e^{x^2+y}$, find $\frac{\delta z}{\delta x}$ and $\frac{\delta z}{\delta y}$.

Given $z = \log(u^2 + v)$, $u = e^{x^2+y^2}$

$v = e^{x^2+y}$

$\therefore$ z is a composite function of x and y

$$\frac{\delta z}{\delta x} = \frac{\delta z}{\delta u}\frac{\delta u}{\delta x} + \frac{\delta z}{\delta v}\frac{\delta v}{\delta x}$$

$$= \frac{1}{u^2+v} \cdot \frac{\delta}{\delta u}(u^2+v) \cdot (e^{x^2+y^2}) \cdot \frac{\delta}{\delta x}(x^2+y^2)$$

$$+ \frac{1}{u^2+v\delta v} \frac{\delta}{\delta v}(u^2+v)(e^{x^2+y}) \cdot \frac{\delta}{\delta x}(x^2+y)$$

$$= \frac{2u}{u^2+v}(e^{x^2+y^2})(2x) + \frac{1}{u^2+v}(1)(e^{x^2+y})(2x)$$

$$= \frac{4ux}{u^2+v} e^{x^2+y^2} + \frac{2x}{u^2+v} e^{x^2+y}$$

$$= \frac{4ux}{u^2+v} \cdot u + \frac{2x}{u^2+v} v = \frac{4u^2x}{u^2+v} + \frac{2vx}{u^2+v}$$

$$= \frac{2x(2u^2+v)}{u^2+v}$$

Now $\frac{\delta z}{\delta y} = \frac{\delta z}{\delta u} \cdot \frac{\delta u}{\delta y} + \frac{\delta z}{\delta u} \frac{\delta v}{\delta y}$

$$= \frac{1}{u^2+v} \frac{\delta}{\delta u}(u^2+v) e^{x^2+y^2} \frac{\delta}{\delta y}(x^2+y^2)$$

$$+ \frac{1}{u^2+v} \frac{\delta}{\delta v}(u^2+v) e^{x^2+y} \frac{\delta}{\delta y}(x^2+y)$$

$$= \frac{1}{u^2+v} 2u\, e^{x^2+y}(2y) + \frac{1}{u^2+v}(1) e^{x^2+y} \cdot 1$$

$$= \frac{4uyu}{u^2+v} + \frac{v}{u^2+v} = \frac{4u^2y+v}{(u^2+v)}$$

PART - C

Q Find the maximum and minimum distances of the point (3,4,12) from the sphere $x^2+y^2+z^2=1$

Sol The Given sphere is $x^2+y^2+z^2=1$ —— (1)

Let $P(x,y,z)$ be any point on the sphere.

Given point is $Q(3,4,12)$

$\therefore$ distance $|PQ| = \sqrt{(x-3)^2+(y-4)^2+(z-12)^2}$

$f(x,y,z) = (PQ)^2 = (x-3)^2+(y-4)^2+(z-12)^2$

and Given Condition is

$\phi(x,y,z) = x^2+y^2+z^2-1=0$ —— (2)

Consider Lagrange's function

$F(x,y,z) = f(x,y,z) + \lambda\phi(x,y,z)$

$= (x-3)^2+(y-4)^2+(z-12)^2+\lambda(x^2+y^2+z^2-1)$

For stationary values

$\frac{\delta F}{\delta x} = 0 \Rightarrow 2(x-3)+2\lambda x=0$ —— (3)

$\frac{\delta F}{\delta y} = 0 \Rightarrow 2(y-4)+2\lambda y=0$ —— (4)

$\frac{\delta F}{\delta y} = 0 \Rightarrow 2(z-12)+2\lambda z=0$ —— (5)

Multiplying (3) by x, (4) by y, (5) by z

and adding, we get

$$2(x^2+y^2+z^2) - 6x - 8y - 24z + 2\lambda(x^2+y^2+z^2) = 0$$

$$2 - 6x - 8y - 24z + 2\lambda = 0 \quad [\because x^2+y^2+z^2 = 1]$$

$$3x + 4y + 12z = 1 + \lambda \quad \text{——— (6)}$$

From (3), (4), and (5),

$$x = \frac{3}{1+\lambda}, \; y = \frac{4}{1+\lambda}, \; z = \frac{12}{1+\lambda} \quad \text{——— (7)}$$

Putting these values of x, y, z in (6), we have

$$\frac{9}{1+\lambda} + \frac{16}{1+\lambda} + \frac{144}{1+\lambda} = 1+\lambda$$

or $(1+\lambda)^2 = 169$

$$1+\lambda = \pm 13$$

When $\lambda = 12$, from (7), we get

$$x = \frac{3}{13}, \; y = \frac{4}{13}, \; z = \frac{12}{13}$$

When $\lambda = -14$ from (7), we get

$$x = -\frac{3}{13}, \; y = -\frac{4}{13}, \; z = -\frac{12}{13}$$

Thus, we get two points $A\left(\frac{3}{13}, \frac{4}{13}, \frac{12}{13}\right)$

and $B\left(-\frac{3}{13}, -\frac{4}{13}, -\frac{12}{13}\right)$ on the sphere which are at a maximum or minimum distance from the given point Q.

$$QA = \sqrt{\left(3-\frac{3}{13}\right)^2+\left(4-\frac{4}{13}\right)^2+\left(12-\frac{12}{13}\right)^2} = 12$$

$$QB = \sqrt{\left(3+\frac{3}{13}\right)^2+\left(4+\frac{4}{13}\right)^2+\left(12+\frac{12}{13}\right)^2} = 14$$

$\therefore$ minimum distance is at point

$A\left(\frac{3}{13}, \frac{4}{13}, \frac{12}{13}\right)$.

Maximum distance is at point

$B\left(-\frac{3}{13}, -\frac{4}{13}, -\frac{12}{13}\right)$

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